

THE MOUNTAIN

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THE MOUNTAIN. (1913)

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\mathbb{R}^n 中的点 x 到集合 A 的距离定义为 $d(x, A) = \inf_{y \in A} \|x - y\|$ 。若 A 是凸集，则 $d(x, A)$ 是 x 关于 A 的凸函数。证明：若 A 是凸集，则 $d(x, A)$ 是 x 关于 A 的凸函数。

证明：设 $x_1, x_2 \in \mathbb{R}^n$ ， $\lambda \in [0, 1]$ 。记 $x = \lambda x_1 + (1-\lambda)x_2$ 。由距离的定义，存在 $y_1 \in A$ 使得 $d(x_1, A) = \|x_1 - y_1\|$ ，存在 $y_2 \in A$ 使得 $d(x_2, A) = \|x_2 - y_2\|$ 。由于 A 是凸集，故 $\lambda y_1 + (1-\lambda)y_2 \in A$ 。于是有：

$$d(x, A) \leq \|x - (\lambda y_1 + (1-\lambda)y_2)\| = \|\lambda(x_1 - y_1) + (1-\lambda)(x_2 - y_2)\|$$

$$\leq \lambda \|x_1 - y_1\| + (1-\lambda) \|x_2 - y_2\| = \lambda d(x_1, A) + (1-\lambda) d(x_2, A)$$

因此 $d(x, A)$ 是 x 关于 A 的凸函数。

设 $f: \mathbb{R}^n \rightarrow \mathbb{R}$ 是凸函数， $A \subseteq \mathbb{R}^n$ 是凸集。证明：若 f 在 A 上可微，则 f 在 A 上的局部极小值点必是全局极小值点。

证明：设 x^* 是 f 在 A 上的局部极小值点。由于 f 在 x^* 处可微，故存在 $\nabla f(x^*) = 0$ 。对于任意 $x \in A$ ，由凸函数的性质，有：

$$f(x) \geq f(x^*) + \nabla f(x^*)^T (x - x^*) = f(x^*)$$

因此 x^* 是 f 在 A 上的全局极小值点。

设 $f: \mathbb{R}^n \rightarrow \mathbb{R}$ 是凸函数， $A \subseteq \mathbb{R}^n$ 是凸集。证明：若 f 在 A 上可微，且 x^* 是 f 在 A 上的局部极小值点，则 $\nabla f(x^*) = 0$ 。

证明：设 x^* 是 f 在 A 上的局部极小值点。对于任意 $x \in A$ ，由凸函数的性质，有：

$$f(x) \geq f(x^*) + \nabla f(x^*)^T (x - x^*)$$

令 $x = x^* + \epsilon v$ ，其中 $v \in \mathbb{R}^n$ 是任意向量， $\epsilon > 0$ 。由于 A 是凸集，故 $x \in A$ 。于是有：

$$f(x^* + \epsilon v) \geq f(x^*) + \nabla f(x^*)^T (\epsilon v)$$

由于 x^* 是局部极小值点，故 $f(x^* + \epsilon v) \geq f(x^*)$ 。因此有：

$$\nabla f(x^*)^T v \geq 0$$

同理，令 $x = x^* - \epsilon v$ ，可得 $\nabla f(x^*)^T v \leq 0$ 。因此 $\nabla f(x^*)^T v = 0$ 。由于 v 是任意向量，故 $\nabla f(x^*) = 0$ 。

11.

$$f_1(x) = 2x^2 + 3x + 1, f_2(x) = 3x^2 + 2x + 1, f_3(x) = 4x^2 + 3x + 1, f_4(x) = 5x^2 + 4x + 1, \dots$$

$$\Delta f_1(x) = 4x + 3, \Delta f_2(x) = 6x + 4, \Delta f_3(x) = 8x + 5, \Delta f_4(x) = 10x + 6, \dots$$

where $\Delta f(x) = f(x+1) - f(x)$.

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ be a polynomial of degree n . Then $\Delta f(x)$ is a polynomial of degree $n-1$. In particular, $\Delta^2 f(x)$ is a polynomial of degree $n-2$, $\Delta^3 f(x)$ is a polynomial of degree $n-3$, and so on. Thus, $\Delta^n f(x)$ is a constant polynomial, and $\Delta^{n+1} f(x) = 0$.

$$\Delta^n f(x) = n! a_n, \Delta^{n+1} f(x) = 0, \Delta^{n+2} f(x) = 0, \dots$$

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ be a polynomial of degree n . Then $\Delta^n f(x)$ is a constant polynomial, and $\Delta^{n+1} f(x) = 0$.

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ be a polynomial of degree n . Then $\Delta^n f(x)$ is a constant polynomial, and $\Delta^{n+1} f(x) = 0$. In particular, $\Delta^n f(x) = n! a_n$, and $\Delta^{n+1} f(x) = 0$.

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$\frac{1}{2} \frac{d}{dt} (x^2 + y^2) = \frac{1}{2} (2x \frac{dx}{dt} + 2y \frac{dy}{dt}) = x \frac{dx}{dt} + y \frac{dy}{dt}$
 $\frac{1}{2} \frac{d}{dt} (x^2 + y^2) = \frac{1}{2} \frac{d}{dt} (r^2) = \frac{1}{2} (2r \frac{dr}{dt}) = r \frac{dr}{dt}$
 $x \frac{dx}{dt} + y \frac{dy}{dt} = r \frac{dr}{dt}$
 $\frac{1}{2} \frac{d}{dt} (x^2 + y^2) = r \frac{dr}{dt}$
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Let T_1, T_2, \dots, T_n be independent events.

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Let $A = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(A) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $B = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(B) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $C = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(C) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $D = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(D) = P(T_1)P(T_2)\dots P(T_n)$.

Let $E = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(E) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $F = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(F) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $G = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(G) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $H = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(H) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $I = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(I) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $J = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(J) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $K = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(K) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $L = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(L) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $M = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(M) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $N = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(N) = P(T_1)P(T_2)\dots P(T_n)$.

Let $O = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(O) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $P = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(P) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $Q = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(Q) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $R = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(R) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $S = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(S) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $T = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(T) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $U = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(U) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $V = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(V) = P(T_1)P(T_2)\dots P(T_n)$.
 Let $W = T_1 \cup T_2 \cup \dots \cup T_n$. Then $P(W) = 1 - P(\text{none of } T_1, T_2, \dots, T_n \text{ occur})$.
 Let $X = T_1 \cap T_2 \cap \dots \cap T_n$. Then $P(X) = P(T_1)P(T_2)\dots P(T_n)$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$.

1. Compute $f(2)$ and $f(3)$. Then compute $f(f(2))$ and $f(f(3))$.

2. Compute $f(1)$ and $f(2)$. Then compute $f(f(1))$ and $f(f(2))$.

3. Compute $f(0)$ and $f(1)$. Then compute $f(f(0))$ and $f(f(1))$.

4. Compute $f(-1)$ and $f(0)$. Then compute $f(f(-1))$ and $f(f(0))$.

5. Compute $f(-2)$ and $f(-1)$. Then compute $f(f(-2))$ and $f(f(-1))$.

6. Compute $f(-3)$ and $f(-2)$. Then compute $f(f(-3))$ and $f(f(-2))$.

7. Compute $f(-4)$ and $f(-3)$. Then compute $f(f(-4))$ and $f(f(-3))$.

8. Compute $f(-5)$ and $f(-4)$. Then compute $f(f(-5))$ and $f(f(-4))$.

9. Compute $f(-6)$ and $f(-5)$. Then compute $f(f(-6))$ and $f(f(-5))$.

10. Compute $f(-7)$ and $f(-6)$. Then compute $f(f(-7))$ and $f(f(-6))$.

11. Compute $f(-8)$ and $f(-7)$. Then compute $f(f(-8))$ and $f(f(-7))$.

12. Compute $f(-9)$ and $f(-8)$. Then compute $f(f(-9))$ and $f(f(-8))$.

13. Compute $f(-10)$ and $f(-9)$. Then compute $f(f(-10))$ and $f(f(-9))$.

14. Compute $f(-11)$ and $f(-10)$. Then compute $f(f(-11))$ and $f(f(-10))$.

15. Compute $f(-12)$ and $f(-11)$. Then compute $f(f(-12))$ and $f(f(-11))$.

16. Compute $f(-13)$ and $f(-12)$. Then compute $f(f(-13))$ and $f(f(-12))$.

17. Compute $f(-14)$ and $f(-13)$. Then compute $f(f(-14))$ and $f(f(-13))$.

18. Compute $f(-15)$ and $f(-14)$. Then compute $f(f(-15))$ and $f(f(-14))$.

19. Compute $f(-16)$ and $f(-15)$. Then compute $f(f(-16))$ and $f(f(-15))$.

20. Compute $f(-17)$ and $f(-16)$. Then compute $f(f(-17))$ and $f(f(-16))$.

21. Compute $f(-18)$ and $f(-17)$. Then compute $f(f(-18))$ and $f(f(-17))$.

22. Compute $f(-19)$ and $f(-18)$. Then compute $f(f(-19))$ and $f(f(-18))$.

23. Compute $f(-20)$ and $f(-19)$. Then compute $f(f(-20))$ and $f(f(-19))$.

■ $x^2 + 1 = (x + 1)(x + 1) = (x + 1)^2$. This is the same as the equation $x^2 + 1 = 2x + 1$.
 ■ $x^2 + 2x + 1 = (x + 1)(x + 1) = (x + 1)^2$. This is the same as the equation $x^2 + 2x + 1 = 2x + 1$.
 ■ $x^2 + 4x + 4 = (x + 2)(x + 2) = (x + 2)^2$. This is the same as the equation $x^2 + 4x + 4 = 2x + 2$.
 ■ $x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$. This is the same as the equation $x^2 + 6x + 9 = 2x + 3$.
 ■ $x^2 + 8x + 16 = (x + 4)(x + 4) = (x + 4)^2$. This is the same as the equation $x^2 + 8x + 16 = 2x + 4$.
 ■ $x^2 + 10x + 25 = (x + 5)(x + 5) = (x + 5)^2$. This is the same as the equation $x^2 + 10x + 25 = 2x + 5$.
 ■ $x^2 + 12x + 36 = (x + 6)(x + 6) = (x + 6)^2$. This is the same as the equation $x^2 + 12x + 36 = 2x + 6$.
 ■ $x^2 + 14x + 49 = (x + 7)(x + 7) = (x + 7)^2$. This is the same as the equation $x^2 + 14x + 49 = 2x + 7$.
 ■ $x^2 + 16x + 64 = (x + 8)(x + 8) = (x + 8)^2$. This is the same as the equation $x^2 + 16x + 64 = 2x + 8$.
 ■ $x^2 + 18x + 81 = (x + 9)(x + 9) = (x + 9)^2$. This is the same as the equation $x^2 + 18x + 81 = 2x + 9$.
 ■ $x^2 + 20x + 100 = (x + 10)(x + 10) = (x + 10)^2$. This is the same as the equation $x^2 + 20x + 100 = 2x + 10$.
 ■ $x^2 + 22x + 121 = (x + 11)(x + 11) = (x + 11)^2$. This is the same as the equation $x^2 + 22x + 121 = 2x + 11$.
 ■ $x^2 + 24x + 144 = (x + 12)(x + 12) = (x + 12)^2$. This is the same as the equation $x^2 + 24x + 144 = 2x + 12$.
 ■ $x^2 + 26x + 169 = (x + 13)(x + 13) = (x + 13)^2$. This is the same as the equation $x^2 + 26x + 169 = 2x + 13$.
 ■ $x^2 + 28x + 196 = (x + 14)(x + 14) = (x + 14)^2$. This is the same as the equation $x^2 + 28x + 196 = 2x + 14$.
 ■ $x^2 + 30x + 225 = (x + 15)(x + 15) = (x + 15)^2$. This is the same as the equation $x^2 + 30x + 225 = 2x + 15$.
 ■ $x^2 + 32x + 256 = (x + 16)(x + 16) = (x + 16)^2$. This is the same as the equation $x^2 + 32x + 256 = 2x + 16$.
 ■ $x^2 + 34x + 289 = (x + 17)(x + 17) = (x + 17)^2$. This is the same as the equation $x^2 + 34x + 289 = 2x + 17$.
 ■ $x^2 + 36x + 324 = (x + 18)(x + 18) = (x + 18)^2$. This is the same as the equation $x^2 + 36x + 324 = 2x + 18$.
 ■ $x^2 + 38x + 361 = (x + 19)(x + 19) = (x + 19)^2$. This is the same as the equation $x^2 + 38x + 361 = 2x + 19$.
 ■ $x^2 + 40x + 400 = (x + 20)(x + 20) = (x + 20)^2$. This is the same as the equation $x^2 + 40x + 400 = 2x + 20$.
 ■ $x^2 + 42x + 441 = (x + 21)(x + 21) = (x + 21)^2$. This is the same as the equation $x^2 + 42x + 441 = 2x + 21$.
 ■ $x^2 + 44x + 484 = (x + 22)(x + 22) = (x + 22)^2$. This is the same as the equation $x^2 + 44x + 484 = 2x + 22$.
 ■ $x^2 + 46x + 529 = (x + 23)(x + 23) = (x + 23)^2$. This is the same as the equation $x^2 + 46x + 529 = 2x + 23$.
 ■ $x^2 + 48x + 576 = (x + 24)(x + 24) = (x + 24)^2$. This is the same as the equation $x^2 + 48x + 576 = 2x + 24$.
 ■ $x^2 + 50x + 625 = (x + 25)(x + 25) = (x + 25)^2$. This is the same as the equation $x^2 + 50x + 625 = 2x + 25$.
 ■ $x^2 + 52x + 676 = (x + 26)(x + 26) = (x + 26)^2$. This is the same as the equation $x^2 + 52x + 676 = 2x + 26$.
 ■ $x^2 + 54x + 729 = (x + 27)(x + 27) = (x + 27)^2$. This is the same as the equation $x^2 + 54x + 729 = 2x + 27$.
 ■ $x^2 + 56x + 784 = (x + 28)(x + 28) = (x + 28)^2$. This is the same as the equation $x^2 + 56x + 784 = 2x + 28$.
 ■ $x^2 + 58x + 841 = (x + 29)(x + 29) = (x + 29)^2$. This is the same as the equation $x^2 + 58x + 841 = 2x + 29$.
 ■ $x^2 + 60x + 900 = (x + 30)(x + 30) = (x + 30)^2$. This is the same as the equation $x^2 + 60x + 900 = 2x + 30$.
 ■ $x^2 + 62x + 961 = (x + 31)(x + 31) = (x + 31)^2$. This is the same as the equation $x^2 + 62x + 961 = 2x + 31$.
 ■ $x^2 + 64x + 1024 = (x + 32)(x + 32) = (x + 32)^2$. This is the same as the equation $x^2 + 64x + 1024 = 2x + 32$.
 ■ $x^2 + 66x + 1089 = (x + 33)(x + 33) = (x + 33)^2$. This is the same as the equation $x^2 + 66x + 1089 = 2x + 33$.
 ■ $x^2 + 68x + 1156 = (x + 34)(x + 34) = (x + 34)^2$. This is the same as the equation $x^2 + 68x + 1156 = 2x + 34$.
 ■ $x^2 + 70x + 1225 = (x + 35)(x + 35) = (x + 35)^2$. This is the same as the equation $x^2 + 70x + 1225 = 2x + 35$.
 ■ $x^2 + 72x + 1296 = (x + 36)(x + 36) = (x + 36)^2$. This is the same as the equation $x^2 + 72x + 1296 = 2x + 36$.
 ■ $x^2 + 74x + 1369 = (x + 37)(x + 37) = (x + 37)^2$. This is the same as the equation $x^2 + 74x + 1369 = 2x + 37$.
 ■ $x^2 + 76x + 1444 = (x + 38)(x + 38) = (x + 38)^2$. This is the same as the equation $x^2 + 76x + 1444 = 2x + 38$.
 ■ $x^2 + 78x + 1521 = (x + 39)(x + 39) = (x + 39)^2$. This is the same as the equation $x^2 + 78x + 1521 = 2x + 39$.
 ■ $x^2 + 80x + 1600 = (x + 40)(x + 40) = (x + 40)^2$. This is the same as the equation $x^2 + 80x + 1600 = 2x + 40$.
 ■ $x^2 + 82x + 1681 = (x + 41)(x + 41) = (x + 41)^2$. This is the same as the equation $x^2 + 82x + 1681 = 2x + 41$.
 ■ $x^2 + 84x + 1764 = (x + 42)(x + 42) = (x + 42)^2$. This is the same as the equation $x^2 + 84x + 1764 = 2x + 42$.
 ■ $x^2 + 86x + 1849 = (x + 43)(x + 43) = (x + 43)^2$. This is the same as the equation $x^2 + 86x + 1849 = 2x + 43$.
 ■ $x^2 + 88x + 1936 = (x + 44)(x + 44) = (x + 44)^2$. This is the same as the equation $x^2 + 88x + 1936 = 2x + 44$.
 ■ $x^2 + 90x + 2025 = (x + 45)(x + 45) = (x + 45)^2$. This is the same as the equation $x^2 + 90x + 2025 = 2x + 45$.
 ■ $x^2 + 92x + 2116 = (x + 46)(x + 46) = (x + 46)^2$. This is the same as the equation $x^2 + 92x + 2116 = 2x + 46$.
 ■ $x^2 + 94x + 2209 = (x + 47)(x + 47) = (x + 47)^2$. This is the same as the equation $x^2 + 94x + 2209 = 2x + 47$.
 ■ $x^2 + 96x + 2304 = (x + 48)(x + 48) = (x + 48)^2$. This is the same as the equation $x^2 + 96x + 2304 = 2x + 48$.
 ■ $x^2 + 98x + 2401 = (x + 49)(x + 49) = (x + 49)^2$. This is the same as the equation $x^2 + 98x + 2401 = 2x + 49$.
 ■ $x^2 + 100x + 2500 = (x + 50)(x + 50) = (x + 50)^2$. This is the same as the equation $x^2 + 100x + 2500 = 2x + 50$.

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Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = x^2 + 2x + 1$.

Find $f(3)$.

Since $f(x) = x^2 + 2x + 1$, we have $f(3) = 3^2 + 2 \cdot 3 + 1 = 9 + 6 + 1 = 16$.
 Therefore, $f(3) = 16$.

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Let $\mathcal{A} = \{A_1, A_2, \dots, A_n\}$ be a collection of sets. Then $\bigcap_{i=1}^n A_i$ is the set of elements that belong to every A_i . Similarly, $\bigcup_{i=1}^n A_i$ is the set of elements that belong to at least one A_i .

De Morgan's Laws state that for any collection of sets \mathcal{A} , the complement of the intersection is the union of the complements, and the complement of the union is the intersection of the complements. In symbols: $\overline{\bigcap_{i=1}^n A_i} = \bigcup_{i=1}^n \overline{A_i}$ and $\overline{\bigcup_{i=1}^n A_i} = \bigcap_{i=1}^n \overline{A_i}$.

The distributive laws of set theory state that intersection distributes over union, and union distributes over intersection. That is, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

The principle of inclusion-exclusion provides a formula for the size of the union of two or more sets. For two sets A and B , the formula is $|A \cup B| = |A| + |B| - |A \cap B|$. For three sets A, B, C , the formula is $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

Counting problems often involve determining the number of elements in a set or the number of ways to choose elements from a set. The binomial coefficient $\binom{n}{k}$ represents the number of ways to choose k elements from a set of n elements. The binomial theorem states that $(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$.

Permutations and combinations are fundamental concepts in counting. A permutation is an arrangement of objects in a specific order, while a combination is a selection of objects without regard to order. The number of permutations of n objects is $n!$, and the number of combinations of n objects taken k at a time is $\binom{n}{k}$.

Let \mathcal{L}_1 and \mathcal{L}_2 be lines in \mathbb{R}^3 . If \mathcal{L}_1 and \mathcal{L}_2 are parallel, then they are either coincident or disjoint. If \mathcal{L}_1 and \mathcal{L}_2 are not parallel, then they intersect at exactly one point.

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$\alpha \in \mathbb{R}$ is a constant, $\alpha \neq 0$. The function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = \alpha x^2 + 2x + 1$. Find the value of α such that $f(x) = 0$ has two distinct real roots.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 2x^2 - 5x + 3$. Find the set of values of x such that $f(x) < 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 4x + 4$. Find the set of values of x such that $f(x) \leq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 6x + 9$. Find the set of values of x such that $f(x) \geq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 8x + 16$. Find the set of values of x such that $f(x) \leq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 10x + 25$. Find the set of values of x such that $f(x) \geq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 12x + 36$. Find the set of values of x such that $f(x) \leq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 14x + 49$. Find the set of values of x such that $f(x) \geq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 16x + 64$. Find the set of values of x such that $f(x) \leq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 18x + 81$. Find the set of values of x such that $f(x) \geq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 20x + 100$. Find the set of values of x such that $f(x) \leq 0$.

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 22x + 121$. Find the set of values of x such that $f(x) \geq 0$.

1. 在 \mathbb{R}^n 中，设 $A = (a_{ij})_{n \times n}$ 为实对称矩阵，且 $A^2 = A$ 。证明：
 (1) A 的特征值为 0 或 1。
 (2) A 可对角化，且存在正交矩阵 Q ，使得 $Q^{-1}AQ = \text{diag}\{\underbrace{1, \dots, 1}_r, \underbrace{0, \dots, 0}_{n-r}\}$ 。
 (3) A 的秩为 r 。

证明：(1) 设 λ 为 A 的特征值， α 为对应的特征向量， $\alpha \neq 0$ 。由 $A^2 = A$ 得 $A^2\alpha = A\alpha$ ，即 $\lambda^2\alpha = \lambda\alpha$ 。因为 $\alpha \neq 0$ ，所以 $\lambda^2 = \lambda$ ，解得 $\lambda = 0$ 或 $\lambda = 1$ 。
 (2) 由 (1) 知， A 的特征值只有 0 和 1。设 $\lambda_1 = 1$ 的特征子空间为 V_1 ， $\lambda_2 = 0$ 的特征子空间为 V_2 。因为 $A^2 = A$ ，所以 $V_1 \perp V_2$ 。又 $V_1 + V_2 = \mathbb{R}^n$ ，所以 $\mathbb{R}^n = V_1 \oplus V_2$ 。取 V_1 的一组标准正交基 $\alpha_1, \dots, \alpha_r$ ， V_2 的一组标准正交基 $\alpha_{r+1}, \dots, \alpha_n$ ，令 $Q = (\alpha_1, \dots, \alpha_n)$ ，则 Q 为正交矩阵，且 $Q^{-1}AQ = \text{diag}\{\underbrace{1, \dots, 1}_r, \underbrace{0, \dots, 0}_{n-r}\}$ 。
 (3) 由 (2) 知， A 的秩为 r 。

2. 设 $A = (a_{ij})_{n \times n}$ 为实对称矩阵，且 $A^2 = A$ 。证明：
 (1) A 的特征值为 0 或 1。
 (2) A 可对角化，且存在正交矩阵 Q ，使得 $Q^{-1}AQ = \text{diag}\{\underbrace{1, \dots, 1}_r, \underbrace{0, \dots, 0}_{n-r}\}$ 。
 (3) A 的秩为 r 。

证明：(1) 设 λ 为 A 的特征值， α 为对应的特征向量， $\alpha \neq 0$ 。由 $A^2 = A$ 得 $A^2\alpha = A\alpha$ ，即 $\lambda^2\alpha = \lambda\alpha$ 。因为 $\alpha \neq 0$ ，所以 $\lambda^2 = \lambda$ ，解得 $\lambda = 0$ 或 $\lambda = 1$ 。
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 (3) 由 (2) 知， A 的秩为 r 。

$\Delta \left(\frac{1}{x} \right) = -\frac{1}{x^2}$

$\Delta \left(\frac{1}{x^2} \right) = \frac{1}{(x+1)^2} - \frac{1}{x^2} = \frac{x^2 - (x+1)^2}{x^2(x+1)^2} = \frac{-2x-1}{x^2(x+1)^2}$

$\Delta \left(\frac{1}{x^3} \right) = \frac{1}{(x+1)^3} - \frac{1}{x^3} = \frac{x^3 - (x+1)^3}{x^3(x+1)^3} = \frac{-3x^2-3x-1}{x^3(x+1)^3}$

$\Delta \left(\frac{1}{x^4} \right) = \frac{1}{(x+1)^4} - \frac{1}{x^4} = \frac{x^4 - (x+1)^4}{x^4(x+1)^4} = \frac{-4x^3-6x^2-4x-1}{x^4(x+1)^4}$

$\Delta \left(\frac{1}{x^5} \right) = \frac{1}{(x+1)^5} - \frac{1}{x^5} = \frac{x^5 - (x+1)^5}{x^5(x+1)^5} = \frac{-5x^4-10x^3-10x^2-5x-1}{x^5(x+1)^5}$

$\Delta \left(\frac{1}{x^6} \right) = \frac{1}{(x+1)^6} - \frac{1}{x^6} = \frac{x^6 - (x+1)^6}{x^6(x+1)^6} = \frac{-6x^5-15x^4-20x^3-15x^2-6x-1}{x^6(x+1)^6}$

$\Delta \left(\frac{1}{x^7} \right) = \frac{1}{(x+1)^7} - \frac{1}{x^7} = \frac{x^7 - (x+1)^7}{x^7(x+1)^7} = \frac{-7x^6-21x^5-35x^4-35x^3-21x^2-7x-1}{x^7(x+1)^7}$

$\Delta \left(\frac{1}{x^8} \right) = \frac{1}{(x+1)^8} - \frac{1}{x^8} = \frac{x^8 - (x+1)^8}{x^8(x+1)^8} = \frac{-8x^7-28x^6-56x^5-56x^4-28x^3-8x^2-1}{x^8(x+1)^8}$

$\Delta \left(\frac{1}{x^9} \right) = \frac{1}{(x+1)^9} - \frac{1}{x^9} = \frac{x^9 - (x+1)^9}{x^9(x+1)^9} = \frac{-9x^8-36x^7-84x^6-84x^5-36x^4-9x^3-1}{x^9(x+1)^9}$

$\Delta \left(\frac{1}{x^{10}} \right) = \frac{1}{(x+1)^{10}} - \frac{1}{x^{10}} = \frac{x^{10} - (x+1)^{10}}{x^{10}(x+1)^{10}} = \frac{-10x^9-45x^8-126x^7-126x^6-45x^5-10x^4-1}{x^{10}(x+1)^{10}}$

$\Delta \left(\frac{1}{x^{11}} \right) = \frac{1}{(x+1)^{11}} - \frac{1}{x^{11}} = \frac{x^{11} - (x+1)^{11}}{x^{11}(x+1)^{11}} = \frac{-11x^{10}-55x^9-165x^8-165x^7-55x^6-11x^5-1}{x^{11}(x+1)^{11}}$

$\Delta \left(\frac{1}{x^{12}} \right) = \frac{1}{(x+1)^{12}} - \frac{1}{x^{12}} = \frac{x^{12} - (x+1)^{12}}{x^{12}(x+1)^{12}} = \frac{-12x^{11}-66x^{10}-231x^9-231x^8-66x^7-12x^6-1}{x^{12}(x+1)^{12}}$

$\Delta \left(\frac{1}{x^{13}} \right) = \frac{1}{(x+1)^{13}} - \frac{1}{x^{13}} = \frac{x^{13} - (x+1)^{13}}{x^{13}(x+1)^{13}} = \frac{-13x^{12}-78x^{11}-273x^{10}-273x^9-78x^8-13x^7-1}{x^{13}(x+1)^{13}}$

U_2 是 V 的 U_1 的补空间

$\Rightarrow \forall v \in V, \exists u_1 \in U_1, u_2 \in U_2, v = u_1 + u_2$
 $\Rightarrow \forall u_1 \in U_1, \exists u_2 \in U_2, u_1 + u_2 = 0$
 $\Rightarrow \forall u_1 \in U_1, \exists u_2 \in U_2, u_2 = -u_1$

$$u_2 = -u_1 \in U_2 \Rightarrow U_2 = \{ -u_1 | u_1 \in U_1 \}$$

$$\Rightarrow U_2 = \{ -u_1 | u_1 \in U_1 \} = \{ -u_1 | u_1 \in U_1 \} = -U_1$$

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