Joint eigenfunctions for the relativistic Calogero-Moser Hamiltonians of hyperbolic type

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Background

The sine-Gordon equation

$$\partial_x^2 \quad \partial_t^2 \quad = \sin \qquad (1)$$

de nes a relativistically invariant eld theory. Depending on how the eld   is interpreted, one can view (1) either as a classical nonlinear evolution equation or as an interacting relativistic quantum eld theory.

A remarkable feature of both the classical and quantum sine-Gordon models is the presence of 'solitons'. In the quantum case this means that particle creation and annihilation is absent, in a collision the set of momenta is conserved, and the scattering operator for a N-particle collision factorizes as a product of all pair scattering operators.

Some thirty years ago this led one of us (S.R.) to ask the following question:

> Do there exist Hamiltonian dynamics for N point particles that lead to the same factorized scattering?

In the classical case this question has been answered in the af rmative. An important aim of the present work is to show that the answer is af rmative also in the quantum case.

A relativistic Calogero-Moser system

The relevant N-particle system is the so-called relativistic Calogero-Moser system of hyperbolic type. In the quantum case this system is given by $2N$ commuting Hamiltonians

$$H_{k;} \quad \sum_{\substack{I \subset \{1;:::;N\} \\ |I|=k}} \prod_{\substack{m \in I \\ n \notin I}} f_{;}\,(x_m \quad x_n) \prod_{l \in I} \exp(\quad ia \quad \partial_{x_l}) \prod_{\substack{m \in I \\ n \notin I}} f_{;+}(x_m \quad x_n);$$

where $k = 1;\ldots;N$, $= +;$, and

$$f_{;}\,(z) = \left(\frac{\sinh \quad (z \quad ib)/a}{\sinh \quad z/a}\right)^{1/2}:$$

Physical picture: For $= +$, there are two length scales, namely

$$a_+ \quad 2 = ; \qquad \text{(imaginary period = interaction length)},$$

and

$$a \quad \hbar/mc; \qquad \text{(shift step size = Compton wavelength)},$$

with $\hbar$ Planck's constant, m<N